General Materials and Supplies: posters with the “Big Ideas”, white boards, chart paper, markers, erasers, handouts #1 - 4, Essential Standards, Big Ideas, NCTM statement cards (printed on card stock from handout #3)

<table>
<thead>
<tr>
<th>(slide)</th>
<th>Tasks/Activity</th>
<th>Personal Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(slide 1) <strong>Partners Title</strong> slide</td>
<td>Welcome and introductions of leaders. Do any paper work needed for the professional development.</td>
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<tr>
<td>(slide 2) <strong>Video Overview</strong></td>
<td>This is the same video that will be part of every grade level’s first module.</td>
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<tr>
<td>(slide 3 – slide 10)</td>
<td>These are introductory slides. Only slide 3 is shown on the left. Do not linger long over these slides. Slides 11 through 18 set the stage for a problem solving approach to teaching. Move through these slides quickly also.</td>
<td></td>
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<tr>
<td>(slide 11) <strong>The Essential Question is</strong>…</td>
<td></td>
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<tr>
<td>• <em>Why are we here?</em></td>
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<tr>
<td>• <em>Talk at your table: What do we want our students to get out of the mathematics courses that they take?</em></td>
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<tr>
<td>Answers will vary.</td>
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<tr>
<td>We want to provide our students (and yours) with thinking skills that they can use after they leave our classes.</td>
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</table>
(slide 12) **We Hope to…**
- Provide meaningful activities for use in your classrooms
- Model appropriate classroom instructional strategies, and
- Engage you in meaningful dialogue and conversation about mathematics and the teaching of mathematics

(sid 13) **Please Keep in Mind**
*There is no one right way to teach problem solving (and it would be presumptuous to consider recommending one).*
What is critical is that students have opportunities to solve multiple complex and simple problems using a variety of strategies.

(slide 14) **Simply put**
*In your teaching, ask yourself: Who is doing the thinking?*
*We believe that students learn how to solve problems by doing and not by watching.*

(slide 15) **Professional Development Goals:**
Use the poster with the “Big Ideas”
Read through the goals and elaborate on each or ask a participant to read each goal.
Ask, “What additional expectations did you bring to the workshop?” (Answers may vary.)
- Present and solve rich problems for classroom use
- Relate these problems to the NC SCOS and the “Big Ideas”
- Provide and discuss relevant journal articles related to each of the modules
- Expand teacher’s knowledge of mathematics
(slide 16) **Relating Professional Development to the Classroom**
Teachers always maintain the goal of making their instruction relevant; this professional development was designed to model strategies for teaching that will:

* Actively engage students in problem solving
* Provide a structure for student success
* Improve student’s perception of their ability to solve problems
* Improve student’s ability to communicate their understanding of mathematics
* Improve student’s understanding of mathematics
* Increase student awareness of the cohesiveness of mathematics.

(slide 17) **Overview of the Six Modules**
While the content of all of the modules integrates mathematics from multiple strands, each module has a particular focus. The professional development consists of six 2-hour modules. The focus of each module is as follows:

1. *Problem Solving*
2. *Communication*
3. *Geometry and Measurement*
4. *Proportional Reasoning*
5. *Data Analysis*
6. *Algebra*

(slide 18) **Module 1**
Presenters should emphasize that the module title gives the focus of problems to be solved during that session. In this module mathematical reasoning and justifying one’s answer are emphasized. Each of the mathematical strands is woven throughout all of the modules.
(slide 19) **Problem Solving**
This is a quote from NCTM’s *Principles and Standards of School Mathematics.*
In real life, problems are not usually limited to a single strand of mathematics, but often require students to apply concepts and procedures from multiple strands. Also, solutions to real problems are not likely to be found through a known algorithm.

- “Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas.” PSSM, 2000
- Different from drill and practice of “naked numbers,” problem solving provides both practice and support for new learning

(slide 20) **Student Perceptions**
- Many students perceive that they are incapable of solving word problems.
- Some students simply skip the word problems in the textbook, assuming that they will NEVER be able to solve them!

Is this limited to only the low performing students?

(slide 21) **Students Need…**
- Problems that are interesting, engaging, and challenging
- Support and encouragement to learn that they can successfully solve problems
- Time for struggling with ideas and attempting different strategies

(slide 22) **Cori the Camel**
(MATERIALS: white boards, chart paper, markers, erasers)
“Cori the camel is 1000 miles from market. She has 3000 bananas. She can carry a maximum of 1000 bananas and eats one banana for each mile traveled. How many bananas can Cori carry to the market?”

Sketch the problem situation on chart paper with input from the group. Note that this problem can be given to students on the first day of class. Cori has to take the bananas across the desert. How can she do it based on the given constraints?
Group work!

(HANDOUT 1 – “Cori the Camel” problem)

Participants should work in groups to determine their best solution to the problem. Allow approximately 10 minutes for groups to solve the problem. Have them put their answers on chart paper or white boards.

Share your solutions

As participants are writing their solutions, decide the order in which you are going to ask them to share. Look for a variety of solution strategies and be cautious of inviting the most sophisticated solutions to be shared first.

One solution:
The greatest number of bananas that our students have delivered to market is 500. A detailed description of how this was accomplished follows:

  Three loads of 1000 bananas were carried 250 miles into the desert. On the first two trips, 250 bananas were consumed on the first leg of the trip; 500 were dropped in the desert; and 250 bananas were consumed on the return trip. On the third 250-mile trip only 250 bananas were consumed. There are now 1750 bananas at the 250-mile mark.

  Then one load of 1000 bananas is carried 250 miles; a total of 500 bananas are consumed and 500 bananas are dropped at the 500-mile mark in the desert. The remaining 750 bananas are carried to the 500-mile mark (250 are eaten). There are now 1000 bananas at the 500-mile mark.

  The final 1000 bananas were carried the remaining 500 miles to market. 500 bananas were consumed and the remaining 500 bananas were delivered to market.

Note: there is a different solution that results in a higher number of bananas making it to market.

Debriefing Cori the Camel:

Be sure to allow ample time for discussion of the solution strategies.

*What mathematics did you use in solving the Cori the Camel problem?*
Arithmetic and logic

*What problem solving strategies did you use in solving the problem?*
Some possible strategies include:

- Draw a picture, diagrams, tables, arithmetic, logic, trial and error
- How are the solutions similar?
- Answers will vary.
- What is unique about some of the solutions?
- Answers will vary.

(slide 26) **Reflecting on Problem Solving**

*Why is it important to share and discuss solutions even when some do not have “correct” answers?*

Cori the Camel is **one** example of a mathematical problem where student explanation is more important than the answer, because once an answer is found then the question becomes, “Is it the best solution?” or “Is it optimal?” which is an important part of mathematical modeling in the real world. With Cori, students are working to find the best solution.

Discuss the important aspects of problem solving.

While there are a variety of important aspects in problem solving, we believe (and hope to convince you) that it is more important that students organize and create a logical plan for Cori and that they are able to explain the reasoning behind their plan.

*What are the important aspects of problem solving?*

- Developing a strategy to solve the problem
- Writing an equation that models the problem
- Explaining everyone’s thinking
- Getting the correct answer to the problem
- Optimizing solutions – determining if there is a “best” solution

(slide 27) **Problem Solving in your Classroom**

*What is the attitude of the students you teach?*

*How can we impact the attitudes of the students in our classrooms?*

When teachers, in both actions and words, communicate that they believe that students can become good problem solvers their students are more likely to attain this goal.
Essential to Developing Problem Solving Skills

In order to teach problem solving, teachers must provide students with real problems, not just tasks for which the students have some applicable algorithm.

In order to teach problem solving, teachers must plan time for thinking and reasoning-incorporating discussions of algorithms and strategies within the context of tasks.

What are Real Problems

Table discussion: What are some “real problems” that you have used in your classroom? Take enough time for participants to share specific problems with the whole group.

Now What?

State the following question, but not for discussion. This is a lead-in slide for Polya’s problem solving steps. These are included in most textbooks.

Now that we have some ideas of problems to solve, how do we teach students to go about solving those problems?

Polya’s Problem Solving Steps

Present and discuss Polya’s steps for problem solving – (Problem Solving in School Mathematics NCTM 1980 yearbook)

1. Understand the problem
2. Devise a plan

On the next slide will be:

3. Carry out the plan
4. Look back

The difference between novice and expert problem solvers is the amount of time that they spend on 1 and 2. Expert problem solvers question all aspects of the problem they working with and
think about the “what ifs” before they even begin to think about how to find a solution. They make connections to previous problems and solutions, and create a plan of attack that reflects their understanding.

(slide 32) **Polya’s Problem Solving Steps**

3. Carry out the plan
4. Look back

Expert problem solvers continue throughout the problem solving process to ask themselves if they think their plan is leading them to a reasonable solution. Once a solution is reached, problem solvers must reason through the combination of problem and solution- is their answer a reasonable solution to the problem? Do they need to attack the problem with an alternative plan?

For example, the group may have come up with more than one number of bananas that Cori could get to market, but then they had to determine if the strategies that lead to those numbers made sense.

(slide 33) **Looking Back…**

Table discussion followed by group sharing –

*In what sense did you use Polya’s steps as you solved the Cori the Camel problem? Did you approach the problem as an “expert” or as a “novice”?

(slide 34) **Problem Solving Strategies**

(Problem Solving in School Mathematics NCTM 1980 yearbook)

Following are a few strategies that teachers and students might use.

1. Using trial and error
2. Making a sketch or drawing a picture
3. Looking for patterns
4. Solving a simpler problem
5. Creating a chart or Table
6. Working backward
7. **Trying a simulation**
Ask participants which of these strategies they used and were there other strategies utilized that are not in this list.

(slide 35) **Student Work Samples**
(HANDOUT #2 – “Cori the Camel” student work)
Look at student work samples.

*What mathematics did the students use?*
*How would you assess/rank the problem solving abilities of each group of students?*
*What misconceptions do the students have?*
Answers will vary.

(slide 36) **Cori the Camel**
*Why is Cori the Camel a good problem for eighth grade students, even though it does not address one specific standard?*

The problem offers opportunities for mathematical reasoning rather than straightforward practice of an algorithm. Some students will use visualization and draw the solution while others will try to apply a more formal, algebraic approach. Explaining ideas and solution paths encourages conversations.

*What would you consider “success” for the students in your classroom on this problem?*
Answers will vary.

(slide 37) **Assessment of Problem Solving**
*Type of rubric or method of grading depends upon the purpose of the activity and when the task occurs within an instructional sequence*

For example, is the task an introduction to problem solving and the use of different strategies for the same problem, or is it a lesson in which learning to work in groups is the emphasis? Is the task a culminating problem?

*Should Cori be graded? Explain “yes” or “no”?*
Answers will vary.
(slide 38) **Assessment of Problem Solving**

(HANDOUT #3 – Analytic Rubric Sample Source: Bright, G. and Joyner, J. (2004). Dynamic Classroom Assessment: Linking Mathematical Understanding to Instruction in Middle Grades and High School, ETA/Cuisenaire.)

*Holistic rubrics assign a single grade for the work: for top grades all aspects of the solution and explanation must be at high levels*

*Analytic rubrics focus on different dimensions of a response: students might earn a high score on mathematical correctness and a lower score for clarity of explanation*

Evaluation methods for problem solving should be flexible.
A holistic approach to scoring students papers gives one grade for the entire work. An analytic rubric grades components of work such as the correctness of the mathematics, appropriateness of solution, and clarity of explanation and justification of the process. There are a variety of rubrics available on the internet.

Distribute sample analytic rubric for brief small group discussion. A traditional grading scale of A, B, C, etc. or 92-100, 85-91, etc. are holistic scoring scales. In other words, one general grade is given for performance as a whole on the task.

Give participants this website as an example of one place to see examples: http://www.missioncollege.org/depts/math/hobbs/rubrics.html
(Note: the next summer’s professional development will address formative assessment in detail.)

(slide 39) **NC 2009 Essential Standards**

(HANDOUT - NC 2009 Essential Standards)

*Where does this (activity/type of problem) fit in the new NC 8th Grade Essential Standards?*

*Where does this (activity/type of problem) fit in the Big Ideas?*

Have participants look for a few minutes at the new 8th Grade Essential Standards and the Big Ideas handouts. Remind them that they are responsible for bringing these to all mathematics professional development sessions.
What is Problem Solving

Ask participants to elaborate on the following definition.
“It (problem solving) is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation.” (S. Krulik and J. Rudnik Problem Solving: A Handbook for Teachers: Allyn and Bacon, INC., 1980)

What is the teacher’s role in problem solving?

(MATERIALS: NCTM statement cards - printed on card stock from Handout #4)
Have participants group themselves into small groups of 4 or 5 and give each group a set of 5 cards; each card has a statement about problem solving from NCTM.
- At your table assign each person a question to read for a small group discussion
- Decide if the group agrees or disagrees with the statements
- If the statement is true, what does it imply about your role as a teacher of problem solving?

In these groups each person takes a turn reading his or her statement and leading a discussion around the two questions
1.) Do you agree with this statement?
2.) If this is true, what does it imply about your role as a teacher of problem solving?
The next five slides display the statements. Show them as the group discusses the statements.

Statement 1: Teacher’s Role

Students’ learning about and through problem solving and their dispositions toward mathematics are shaped by teachers’ instructional decisions and actions.
PSSM, 2000
Ask one group about implications for teaching from this statement.

Statement 2: Teacher’s Role

Teachers motivate students by encouraging communication and collaboration and urging students to seek complete solutions to challenging problems.
PSSM, 2000
<table>
<thead>
<tr>
<th>Statement 3: Teacher’s Role</th>
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<tbody>
<tr>
<td>Teachers can help students become reflective problem solvers by frequently and openly discussing with them the critical aspects of the problem-solving process.</td>
</tr>
<tr>
<td>PSSM, 2000</td>
</tr>
<tr>
<td>Ask one group about implications for teaching from this statement.</td>
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<tr>
<th>Statement 4: Teacher’s Role</th>
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<tr>
<td>Research suggests that an important difference between successful and unsuccessful problem solvers lies in their beliefs about problem solving, about themselves as problem solvers, and about ways to approach solving problems.</td>
</tr>
<tr>
<td>PSSM, 2000</td>
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<tr>
<td>Ask one group about implications for teaching from this statement.</td>
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<tr>
<th>Statement 5: Teacher’s Role</th>
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<tbody>
<tr>
<td>The essence of problem solving is knowing what to do when confronted with unfamiliar problems.</td>
</tr>
<tr>
<td>PSSM, 2000</td>
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<tr>
<td>Ask one group about implications for teaching from this statement.</td>
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<tr>
<th>Camels and Goats – Oh My!</th>
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<tbody>
<tr>
<td>Did you know camels and goats eat their food in two stages and have an even number of toes on each hoof?</td>
</tr>
<tr>
<td>Source: Gunnersden.com</td>
</tr>
<tr>
<td>(This quote leads into the next problem.)</td>
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</table>
Teachers should provide many opportunities for students to use their problem solving skills as well as create a classroom environment in which openness and creativity can occur.

Real problems are not usually solved in one class period. The problem we are about to do has many parts. We do not intend for this problem to be taught in one day or even in one week. The different parts of it could be done at different times throughout the year. As you work through this problem, think about how and when you could use this and how you might extend this with your students.

(48) **Goat on a Rope**
(MATERIALS: Handout #5 “Goat on a Rope” problems 1-6)
Group participants and instruct them to solve problems 1-6 from the “Goat on a Rope” handout. Leave this (slide) up until your groups are ready to share solutions.

(49) **Solution for Problem 1**
Jacob Luke keeps his goat on a chain that is 3 meters long.

1. If the goat is tied to a metal stake in the center of the yard, what is the area of the grass that the goat can reach to eat? Sketch a picture and show your work.

Solution: $9\pi$ m$^2$ (exact value) or 28.2743 m$^2$ (decimal approximation)

Have participants discuss the difference in these two responses.

(50) **Solution for Problem 2**
Sometimes Jacob ties the goat to the corner of a shed that is 5 meters by 4 meters. The 3-meter rope is attached to the base of a wall at ground level. What is the area of grass that the goat can reach? Show and explain your work.

Solution: $6 \frac{3}{4} \pi$ m$^2$ or 21.2058 m$^2$

Note to participants that it is very important that students recognize exact values as valid
responses and are able to calculate using exact values. As students begin to work multistep problems with these “strange” sometimes irrational values, using a decimal approximation early on in calculation can increase the amount of error within an approximate answer.

(slide 51) **Solution for Problem 3**

Suppose the goat was tied at ground level to the center of the 4-meter shed wall. Would the amount of grass the goat can reach be greater than what he could reach when tied to the corner of the shed? Justify your answer.

Solution: $5\pi$ m$^2$

(slide 52) **Solutions to Problem 4**

Jacob bought a new shed with dimensions doubled to 10 meters by 8 meters, and the goat is tied to the corner of the shed.

a) What do you predict will be the area of grass that the goat will eat?

b) Find the actual area.

c) How does the area in part (b) differ from the area of the grass the goat can eat when tied to the shed that measures 5 meters by 4 meters (Question #2)?

Solutions: predictions will vary, $6\frac{3}{4}\pi$ m$^2$, it does not

(slide 53) **Solutions to Problem 5**

If the dimensions of the shed are again 5 meters by 4 meters, but the length of the rope is doubled, what would be the amount of grass the goat can reach? Sketch a picture and show your work.

Solution: $28\frac{1}{4}\pi$ m$^2$

(slide 54) **Solutions to Problem 6**

Suppose the goat is tied to a brand new clothesline pulled tight in a yard with no other barriers. The clothesline is 4 feet high and 16 feet long. The goat’s chain is 5 feet long. What is the amount of grass the goat can reach? Show and explain your work.

Note: do not get bogged down in a discussion about the height of the goat’s neck, this is an example of how mathematical modeling of real world scenarios makes simplifying assumptions.
All along we’ve been assuming the goat is exactly at the end of the rope. It is a good assumption to make.

Solution: $96 + 9 \pi \text{ m}^2$

(slide 55) **Goats in Your Classroom**

_How will you use this activity in your classroom?_

_What are some modifications or extensions that could be made to the problem?_

Answers will vary. One modification would be to change the shape of the shed.

(slide 56) **Challenging Your Students**

(MATERIALS: Handout #5 “Goat on a Rope” Extension problems)

Assign each pair one of the 12 scenarios from “Extensions A”

_Look at the extensions in your handout_

_With your partner discuss a solution strategy for your assigned problem_

_Create a sketch of the scenario and be prepared to talk about how to solve the problem_

Note: Depending on the time remaining, you may have pairs look over the problems without beginning to solve them. You could revisit extensions A and B in module 2 when you are discussion clear communications.

(slide 57) **Please Share Your Problem…**

_And your solution strategies_

Pick one pair to discuss each problem.

(slide 58) **One Last Look: Extension B**

_The goat on a rope_

Have participants read EXTENSIONS part B.

This time there is a fence-obstruction in the way. This will lead to dealing with irrational
numbers in different ways.  

It is likely that there will not be enough time in the presentation of the module to discuss this extension. Choose an appropriate amount of time for your group.

<table>
<thead>
<tr>
<th>Numbers in different ways.</th>
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<tr>
<th>(slide 59) <strong>What Mathematics did You Use?</strong></th>
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<tr>
<td><em>Area formulas, Pythagorean theorem, rational and irrational numbers, quantitative reasoning, other content or processes?</em></td>
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Allow participants to respond to this list and add any items. The discussion can extend back to Cori the Camel.

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<tr>
<th>(slide 60) <strong>Content and Process</strong></th>
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<tr>
<td><em>Notice: These non-routine activities fit both the Essential Standards and the Big Ideas for middle grades mathematics</em></td>
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<tr>
<td><em>Justify: Mathematical reasoning and the ability to make connections is powerful preparation for students for EOG tests</em></td>
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<tr>
<th>(slide 61) <strong>Impact on Students</strong></th>
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<tr>
<td>“There is no other decision that teachers make that has a greater impact on students’ opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics.” Lappan and Briars</td>
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<th>(slide 62) <strong>What is the value of teaching with problems?</strong></th>
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<tr>
<td><em>Talk with Your Partner</em>...</td>
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Note that this is the closing of the module. Time discussions accordingly. Discuss all of the following statements in partners as you scroll through the slides. Tell
participants that the following statements are from John Van de Walle.

(slide 63) **Value of Teaching with Problems**
- Focuses the students' attention on ideas and sense making
- Develops the belief in students that they are capable of doing mathematics and that mathematics makes sense
- Provides ongoing assessment data that can be used to make instructional decisions, help students succeed, and inform parents


(slide 64) **Value of Teaching with Problems**
- Develops “mathematical power”
- Allows an entry point for a wide range of students
- Engages students so that there are fewer discipline problems

(slide 65) **Value of Teaching with Problems**
*Learning is a Lot of Fun!*

(slide 66) **Reflection**
- These materials were developed with success in implementing the new Mathematics Essentials in mind
- Talk at your table about the ways that teaching students to think prepares them for a new curriculum as well as future learning
<table>
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<th>Reflection</th>
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<tr>
<td><em>These materials were developed with success in implementing the new Mathematics Essentials in mind.</em></td>
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<tr>
<td><em>Talk to your teacher about the ways that teaching students to think prepares them for a new curriculum as well as future learning.</em></td>
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<td>Credit slides</td>
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